

Standard Model Higgs Boson Mass from Borderline Metastability of the Vacuum

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Abstract

We have studied imposing the condition that the Standard Model effective Higgs potential should have two approximately degenerate vacua, such that the vacuum we live in is just barely metastable: the one in which we live has a vacuum expectation value of 246 GeV and the other one should have a vacuum expectation value of order the Planck scale. Alone borderline metastability gives, using the experimental top quark mass 173.1 ± 4.6 GeV, the Higgs mass prediction 121.8 ± 11 GeV. The requirement that the second minimum be at the Planck scale already gave the prediction 173 ± 4 GeV for the top quark mass according to our 1995 paper.

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1 Introduction

The four LEP collaborations, ALEPH [1], DELPHI [2], L3 [3] and OPAL [4], have recently reported on the search for the Standard Model (SM) Higgs boson using data collected during 1999-2000. The combined results are consistent with the hypothesis of the production of the SM Higgs boson with a mass around 115 GeV, and an observed excess in the combined data set of 2.9σ [5]. Of course this experimental signal does not give statistically safe evidence for the existence of a Higgs boson with this mass. However if it should turn out that our calculation together with the eventual improved accuracy in the top quark mass measurements finally gives a Higgs mass of 115 GeV rather precisely: then that would not only support our model but also the existence of a real Higgs boson causing the LEP events.

Two of us [6] have already predicted the SM Higgs boson mass using the philosophy of the Multiple Point Principle [7, 8] (MPP) and also predicted the top quark mass using one more assumption. It has been argued that a mild form of locality breaking in quantum gravity due to baby universes [9], say, which are expected to render coupling constants dynamical, leads to the realization of the MPP in Nature: Nature should choose the coupling constants such that several phases can coexist, *i.e.* the vacuum can exist in “degenerate” phases. The MPP implies that, with renormalisation group corrections, the SM should have two minima in the Higgs field effective potential and the values at the two minima should be the same, *i.e.* $V_{\text{eff}}(\phi_{\text{min},1}) = V_{\text{eff}}(\phi_{\text{min},2})$. This condition tells us that the vacuum in which we live is stable: we are “safe” from the danger of vacuum decay, but lie on the borderline for stability. The other assumption was that the second minimum, in which we do not live, has a Higgs field vacuum expectation value (VEV) of the order of the Planck scale, $\langle |\phi_{\text{min},2}| \rangle \approx M_{\text{Planck}}$, *i.e.* we require a strong first-order phase transition between the two vacua from the point of view of a fundamental scale identified with the Planck scale. Using the usual renormalisation group methods, these two assumptions lead to rather precise predictions for the top (pole) quark mass, $M_t = 173 \pm 4$ GeV, and the Higgs boson mass, $M_H = 135 \pm 9$ GeV. Therefore one might say that the MPP prediction for the Higgs mass does “agree” with the combined LEP experimental results within 2.3 standard deviations.

It is the purpose of this paper to develop further the MPP ideas of [6] and investigate whether they might be made to agree better with the LEP data. So let us start by arguing that, with a Higgs mass of 115 GeV and our reluctance to postulate new physics, we are almost driven to MPP in some form: The number of LEP candidate Higgs particle events of mass $M_H \approx 115$ GeV is in agreement with the SM cross section times branching ratio for a SM Higgs particle. It must be admitted however that they are also consistent with SUSY models, but only because the lower bounds on the other (*e.g.* pseudo-scalar) Higgs masses imply that the SUSY Higgs production cross section is close to that of the SM. In this paper we shall assume that the LEP events correspond to the production of a SM Higgs particle. This means that, unless new physics appears below the Planck scale, the effective Higgs potential [10] will have a second minimum falling below the value at its present minimum, in principle signalling the instability of the vacuum in which we live! Then there are only the following three possibilities:

- There is important new physics at the GUT scale (10^{16} GeV) or below.
- We have to stretch the errors in the computation that the borderline for stability should be 135 ± 9 GeV. Since the stretching should be minimal it would be suggested that the Higgs

mass be after all very close to the borderline, *i.e.* it would support the MPP hypothesis of the previous work [6] that the Higgs mass should just lie on the vacuum stability borderline.

- We must accept that the vacuum is *only metastable*. If we could replace the absolute vacuum stability border by the metastability border, we could bring the allowed SM Higgs mass down by about 10 GeV [11]. But also in this case the Higgs mass would have to be close to the border, now the metastability border.

In this work we take seriously the last possibility, and investigate the idea of introducing a modified MPP to predict the SM Higgs boson mass from the requirement that our vacuum has only barely survived early cosmology (call it metastability MPP). We shall ask if there should be reasons that MPP ideas might after all lead to metastability MPP. Further we shall ask, phenomenologically, whether such a metastability MPP fits the data.

Really the main point of the present article is to argue for and present the Higgs and top quark mass relation for an assumption, which we could reasonably call meta-MPP: the various coupling constants and mass parameters in the Lagrangian density are adjusted in such a way that there exist several (in the SM just 2) vacua – having for instance different Higgs field VEVs – so that one/some of them is/are just on the borderline of decay into and getting eaten up by the other vacuum/vacua in the early Big Bang – or perhaps later.

Let us stress that in the present paper, for the purpose of the stability studies of the vacuum, we assume that the SM is in practice valid almost all the way up to the Planck scale¹. In section 5 we shall argue that, with either the MPP or the meta-MPP assumption, it is suggested that any new fields would to a large extent be decoupled. So the assumption that we can use the pure SM up to the Planck scale is likely to be effectively valid, even if it is not completely true in reality.

In the following section, we briefly review the main idea of degenerate vacua – Multiple Point Principle – and the results from our previous paper [6]. In section 3 we discuss the reasons for believing that the Higgs mass should lie on the borderline of metastability for the vacuum. In section 4 we extract the relation between the top quark and Higgs masses from an article by Espinosa and Quirós [11] and present the results for these masses predicted assuming one of them is known from experiment. Some discussion goes into section 5 and finally section 6 contains our conclusions.

2 “Old” prediction of Higgs mass using Multiple Point Principle

In our previous work [6] we applied the MPP assumption to the pure SM, by postulating that the Higgs effective potential V_{eff} has two minima in the radial direction of the Higgs field – really it means two rings of minima in the Mexican hat – and that these have the same energy density.

The relative height in energy density of the two minima $V_{\text{eff}}(\phi_{\text{min},1})$ and $V_{\text{eff}}(\phi_{\text{min},2})$ depends on the SM parameters such as the Higgs mass. When the Higgs mass for say fixed $\phi_{\text{min},1}$ is lowered, the second minimum energy density $V_{\text{eff}}(\phi_{\text{min},2})$ turns out to go down too. Thus, as

¹The new physics in MPP is about what the values of the SM parameters should be, so that it is strictly complementary to the SM, *i.e.* in no way violates its truth.

soon as the Higgs mass is lowered infinitesimally from the value satisfying the MPP, our vacuum becomes formally unstable under decay into the minimum number 2. So the prediction of MPP is that the parameters of the SM must be such that they lie exactly on the vacuum stability curve.

In the previous paper we used the results on the vacuum stability curve from the articles [12, 13, 14] to obtain the MPP prediction for the Higgs mass, taking the other SM parameters from experiment. Using present day data this prediction is $M_H = 135 \pm 9$ GeV. Adding one extra assumption – namely that the second minimum should be at a value of the Higgs field VEV of the order of magnitude of the Planck energy – led us to a surprisingly accurate prediction for the top quark (pole) mass of $M_t = 173 \pm 4$ GeV. This result was obtained [6] by solving the renormalisation group equations numerically for the running of the Higgs self-coupling λ , the top quark Yukawa coupling constant and the three SM fine structure constants as functions of the energy scale. We then used the approximation of the renormalisation group improved potential, by inserting the running couplings into the classical Higgs potential with the scale identified as the Higgs field strength ϕ .

A priori one would have expected that taking the VEV at the second minimum $\phi_{\min,2}$ to be only order of magnitude-wise that of the Planck scale would leave a big uncertainty in the predicted top quark mass. However, by a remarkable coincidence, this uncertainty turns out to be smaller than the experimental uncertainty in the top quark mass. This remarkable accident is due to (1) there being an approximate fixed point behaviour [15], and (2) the top mass approaching this limit from below at an extremely slow rate. In fact a numerical fit to the results from our renormalisation group calculations shows that the deviation of the top quark mass from its approximate infra-red quasi-fixed point limit varies, approximately, as the inverse of the 42nd root of the VEV at the second minimum:

$$M_t \approx M_{t,\text{qfp}} - \frac{C}{\sqrt[42]{\langle |\phi_{\min,2}| \rangle}} , \quad (1)$$

where $M_{t,\text{qfp}}$ denotes the infra-red quasi-fixed point value and C is a constant.

3 Why should the world be on the borderline of vacuum meta-stability?

As the underlying reason for the MPP we could take it that for some mysterious reason: There has to be a physical realization of both minima over comparable amounts of space-time four volume. This “mysterious” requirement is somewhat analogous to the requirement of a microcanonical ensemble, *i.e.* imposing fixed energy rather than a given temperature. Very often such a microcanonical ensemble is forced to contain more than one phase, for example both ice and water should very often be present if a fixed number of water molecules are given a specified energy; in a fixed volume even a vapour phase and thus a triple point can be provoked.

Let us consider whether standard cosmology can lead to such a co-existence of two phases in space-time. For the original version of MPP with exactly degenerate vacua, the early Universe comes out of the Big Bang in our low VEV vacuum, since it has more light particles and thus a lower free energy density $F = U - ST$ than the second, *i.e.* high VEV, vacuum. If now

the high VEV vacuum had a slightly smaller zero temperature energy density, it should in principle be possible for a bubble of this new vacuum – a vacuum bomb one could say – to be produced when the Universe has cooled, which would expand and produce a domain of high VEV vacuum. However this does not seem very likely, because the wall that must separate the two phases becomes so high in energy per unit area that it becomes practically impossible to make transitions when the temperature is no longer within a factor 10^2 or 10^3 from the Planck temperature. So we conclude that, if the Higgs mass is above the metastability border mass of ~ 122 GeV, the low VEV vacuum comes out of the Big Bang and never develops into the high VEV one.

On the other hand, for Higgs masses below the metastability border, the high VEV vacuum is – by definition – produced in the early Universe. Since the high VEV vacuum has the lower energy density in this case, it can of course never return to the low VEV one. It seems that comparable amounts of four space-time volumes for the two vacua could only exist for a Higgs mass just very close to the metastability border. Assuming continuity of the four space-time volume ratio as a function of the Higgs mass, comparable amounts of the two vacua do occur for a Higgs mass equal to the *metastability* bound. This scenario corresponds precisely to our new meta-MPP, as defined in the introduction.

In summary our motivation for preferring the meta-MPP is the belief that both vacua should be realized somewhere or some time; because otherwise how could they both have any physical significance? But, as we have just argued, this is essentially impossible to achieve except for a Higgs mass very close to the metastability border. It must though be admitted that it may be very difficult to estimate in advance how big a difference in the zero temperature vacuum density is needed, if humanity is ever to produce a vacuum bomb with future technology.

4 Results

Espinosa and Quirós [11] have calculated the lower bound on the (pole) Higgs mass, requiring the vacuum we live in not to have already decayed in the Big Bang, and found the following numerical approximation for the metastability bound:

$$M_H/\text{GeV} \geq [2.278 - 4.654(\alpha_s(M_Z) - 0.124)](M_t/\text{GeV}) - 277 \quad , \quad (2)$$

which is valid for $60 \text{ GeV} < M_H < 125 \text{ GeV}$. We take a theoretical uncertainty [16] in this formula for M_H of $\pm 4.6 \text{ GeV}$, corresponding to an uncertainty of $\pm 2 \text{ GeV}$ in M_t .

The present values reported by the Particle Data Group [17] $\alpha_s(M_Z) = 0.1185(20)$ and $M_t = 174.3 \pm 5.1 \text{ GeV}$, coming from the direct observation of top quark events, give rise via the meta-MPP to the Higgs mass prediction, $M_H = 124 \pm 13 \text{ GeV}$. If one instead used the indirect value of the top quark mass $168.2^{+9.6}_{-7.4} \text{ GeV}$, coming from the SM electroweak fit, our meta-MPP prediction of the Higgs mass becomes $M_H = 110.5^{+23}_{-18} \text{ GeV}$. However combining all the information to fit the top quark mass in the SM, the Particle Data Group gets $M_t = 172.9 \pm 4.6 \text{ GeV}$ assuming a fixed Higgs mass of $98^{+57}_{-38} \text{ GeV}$. Now every unit in the logarithm of the Higgs mass is correlated to increasing the indirectly measured top quark mass by 7.5 GeV . If we thus contemplate a 15 GeV larger Higgs mass than the 100 GeV mass used, the indirect top mass should be increased by 1.06 GeV .

Since in the averaging of indirect and direct top masses, one uses the weight $5.1^2/(5.1^2 + 9.6^2) = 0.22$ for the indirect mass, such an increase by 1.06 GeV shifts the average up by $1.06 \text{ GeV} \cdot 0.22 = 0.23 \text{ GeV}$. In other words the Particle Data Group fitted top quark mass value $M_t = 172.9 \pm 4.6 \text{ GeV}$ gets increased to $173.1 \pm 4.6 \text{ GeV}$, which is thus the best value to use. Our corresponding best present prediction for the Higgs mass is

$$M_H = 121.8 \pm 11 \text{ GeV} . \quad (3)$$

If LEP has indeed determined the Higgs mass to be $M_H = 115_{-0.9}^{+1.3} \text{ GeV}$, the uncertainty in its mass is much less than the $\pm 11 \text{ GeV}$ uncertainty of our prediction. We should then rather use the LEP Higgs mass to predict the top quark mass:

$$M_t = 170.2 \pm 2.0 \text{ GeV} . \quad (4)$$

5 Discussion

Since phenomenologically the meta-MPP seems to be in good agreement with data, we should like to present some further ideas supporting the two main ingredients in our calculation: the meta-MPP itself and essentially the pure SM being valid up to the Planck scale.

5.1 Why MPP or meta-MPP?

If one writes down just randomly a Lagrangian density, without caring for whether the Hamiltonian should be bounded from below, one would probably hit a Lagrangian density providing us with tachyons and negative $\lambda |\phi|^4$ terms so that there would be no bottom to the Hamiltonian at all! So one might ask: How did it come about that there *is* a bottom – seemingly at least – in the energy density? An immediately suggestive answer would be that this has to happen otherwise the vacuum would decay forever. But if this kind of answer should be taken seriously, one should think that the cosmological development involved the decay of many candidate vacua until finally – almost accidentally – an approximate effective Hamiltonian with a bottom occurs. So this type of thinking would suggest that the Universe and the present vacuum may not at all have reached the true bottom of the energy density, but only an “accidental” metastable state on the way.

An alternative answer to the question of the origin of the bottom in the energy density could be that the form of the Lagrangian density, in terms of the parameters/couplings not already fixed by some symmetry, is such that there is a guaranteed bottom. The suggestive example of a symmetry set up along this line is SUSY. In SUSY there is automatically a guaranteed bottom in the energy density, because the Hamiltonian has the form $H = Q^\dagger Q$. The value of the energy density at the bottom is even zero, as is also the phenomenological cosmological constant to high accuracy.

The cosmological constant being nearly zero suggests an idea about how to “derive” MPP: Whatever is the reason for getting a bottom at zero energy density, as the cosmological constant problem suggests, it should have a high chance of giving several degenerate minima. If it can give one zero, why not several of them? We are indebted to Susskind [18] for this argument supporting the MPP. SUSY is actually an example of one of the models that can solve the cosmological

constant problem, but only *if SUSY were an exact symmetry*. As we would generally expect, it actually usually predicts² the MPP. Namely, in SUSY, it is well-known that there are usually flat directions or several zero-energy density minima in the effective potential for the scalars.

5.2 Why essentially the pure SM?

The above calculations were made assuming that the SM really is valid all the way up to the Planck scale. This is perhaps not so realistic in the light of the evidence for finite neutrino masses, suggesting a “new” scale at for instance the see-saw scale of around 10^{12} GeV; or perhaps at some very low scale, but at least a new scale, neither Planck nor weak will do. We can, however, give a very general – but not perfectly functioning – argument that the MPP, which is our main assumption, will tend to adjust the coupling constants so as to favour there being several separate “sectors” containing fields/particle types which *only interact weakly with the other sectors*. For this purpose we should make use of the following formulation of the MPP principle: The MPP (meta or full stability version) means that, by arranging the sizes of the various coupling constants and mass parameters in the Lagrangian density, there should, at least approximately, be made many degenerate minima (vacua). In the metastable case the minima are only approximately degenerate and transitions may take place between them but, by adjustment of the couplings *etc.* they are made to be just on the borderline for decay from one vacuum to another one.

The general argument from MPP in favour of there being approximately decoupled sectors goes like this: In order to get say n degenerate vacua, there is a need for the fine-tuning of $n - 1$ coupling-parameters. We are not caring here for the cosmological constant problem that really all the vacua should have zero energy density, but rather only ask for them all to have the same energy density. Now imagine that there are some proposals for separable “sectors”, in the sense that there are some groups of fields for which there are relatively few terms in the Lagrangian involving fields from both “sectors”. Suppose, for instance, that there are two separable “sectors” and there are just p interaction terms involving fields from both sectors. Suppose further that the MPP-machinery (whatever the physics behind it may be) arranges the p interaction terms between the sectors to be zero and the now separate section 1 to have n_1 (approximately) degenerate minima, while the sector 2 has n_2 degenerate minima. Then the whole theory will actually have gotten $n_1 n_2$ approximately degenerate vacua, because you have all the combinations of one sector 1 vacuum with one sector 2 vacuum. This arrangement would cost the fine-tuning of

$$p + (n_1 - 1) + (n_2 - 1) \text{ parameters} . \quad (5)$$

If, however, the MPP-machinery does not choose the possibility of decoupling by putting the p interaction parameters to zero, then the same number of approximately degenerate vacua, namely $n_1 n_2$, would cost the fine-tuning of

$$n - 1 = (n_1 n_2 - 1) \text{ parameters} . \quad (6)$$

If $p + (n_1 - 1) + (n_2 - 1)$ is smaller than $n_1 n_2 - 1$ it will pay better to decouple the two sectors. In that case one would therefore expect that the MPP will arrange the decoupling, in order to get the biggest number of degenerate vacua.

²However, the MPP derived from SUSY of course does not give any interesting predictions for the values of the SUSY parameters.

Thus if new physics is sufficiently isolated, by there not being so many interaction possibilities with say the SM particles, then the MPP is expected to arrange the few left over interactions to get zero couplings. So we expect that the decoupling gets almost total, or rather as decoupled as can be made. This means that, even if there is some new physics, it is likely that the MPP will adjust the coupling constants so that the interaction between the new physics particles and the SM ones is very often fine-tuned to zero. Therefore it is likely that the new physics can actually be safely ignored, in calculations involving the SM particles. Especially the calculation of the Higgs mass *etc.* constraints, from the requirement of degenerate minima in the SM, will give very closely the same results as if one calculated it in the full model, provided the decoupling takes place as described.

An obvious example of such a separate sector, which may decouple approximately, would be one or more right-handed neutrinos getting their masses from a new Higgs field with its own characteristic mass scale, let us call it ϕ_{B-L} and assign it a gauged $B-L$ charge. In this case, the potentially separated sector consists of the Higgs field ϕ_{B-L} , one or more right handed neutrinos and a gauge field coupled to the $B-L$ charge. Since this gauge field couples to $B-L$, meaning the baryon number minus the lepton number, it will of course thereby be forced to interact also with those SM particles which carry baryon number or lepton number. But the SM (Weinberg-Salam) Higgs field, $\phi = \phi_{WS}$, does not carry any baryon or lepton number, so a $B-L$ gauge field would only influence the SM effective Higgs potential *indirectly*. However a coupling corresponding to the term $\lambda_{\text{int}} |\phi_{B-L}|^2 |\phi_{WS}|^2$, causing a direct interaction between the two Higgs fields, is allowed in the Lagrangian density. It will be the most important term for shifting the $n_1 n_2$ minima away from their energy density values which would be obtained if the proposed “see-saw” sector and the SM sector were indeed decoupled. The above argument should therefore mean that the MPP will make the coupling constant λ_{int} for this interaction term very small or zero.

So the see-saw scale will in first approximation not influence our calculation. However if there were a $B-L$ coupling gauge field, as alluded to, it would contribute to the running of the top quark Yukawa coupling constant g_t making its beta function β_{g_t} more negative in the range above the see-saw scale. This would give a small correction to Eq. (2), which deserves further investigation.

6 Conclusion

We have made what we can consider as a correction to the physical detail of the work by two of us in 1995 on the MPP (Multiple Point Principle) prediction of the SM Higgs mass. The point is that we no longer consider it necessary that a principle roughly like the MPP should be valid in the strict sense of the exact degeneracy of the different minima.

Rather we now consider it more reasonable to assume a modified version, meta-MPP, with approximately the same conclusion as the exact degenerate vacuum energy density version of the MPP. The proposal for guiding us into this version of the MPP could be the requirement that, in the cosmological development of the Universe, four space-time regions having volumes of the same order of magnitude should be realized for the two different minima in the SM effective Higgs potential; and with such a requirement there is only a good chance for realizing it if the coupling constants and the Higgs mass are adjusted to be on the metastability borderline.

The old (exact degeneracy) MPP would not realize the high Higgs field VEV vacuum at all, because the high temperature in the early Big Bang would ensure that the low VEV vacuum – in which we live – is the only one realized. Metastability MPP (meta-MPP) says that the Higgs mass is just on the borderline of Big Bang metastability for our vacuum (the 246 GeV Higgs VEV minimum in the effective potential). Our main point then is that, by replacing the 1995 (exact degeneracy) version of the MPP by the metastability version, we are led to a Higgs mass prediction of $M_H = 121.8 \pm 11$ GeV in agreement (with 0.6σ deviation) with LEP observations.

We want to stress that one can consider our present prediction as a *correction* of the older one [6], $M_H|_{\text{old}} = 135 \pm 9$ GeV. The old MPP version suffers from the fact that the high Higgs VEV vacuum is very hard to realize. Speculative explanations for MPP have trouble in giving physical significance to a vacuum that is never realized.

In the “old” work we also derived the top quark mass – taking the other SM couplings from experiment – by requiring the second minimum in the effective potential to occur for a Higgs field VEV of the Planck energy size – order of magnitude-wise. In this way, we obtained the very successful top quark mass prediction of 173 GeV, with the surprisingly small uncertainty of the order of ± 4 GeV. Preliminary estimates suggest that, when this prediction is corrected according to the modified (metastability) MPP, the value for the top quark mass is reduced by about 1 GeV. Such a reduction in the top quark mass would also cause about a 2 GeV reduction in our 122 GeV Higgs mass prediction (to 120 GeV).

If the meta-MPP picture is correct, our predictions for the near future are:

- The LEP Higgs events will be confirmed.
- The top quark mass will turn out to be on the low mass side but within one standard deviation of its present experimental value. Namely, with the present calculational accuracy and using the 115 GeV LEP Higgs mass, we predict $M_t = 170.2 \pm 2$ GeV.

In conclusion we would claim that our meta-MPP, which is after all a very simple principle, agrees rather well with the LEP candidate Higgs mass.

Note: In a recent preprint [16] Isidori *et al.* give a discussion of the (meta) stability of the vacuum, in the light of the potentially found Higgs particle. They neglect the effects of standard cosmology during the first second in the early Universe and concentrate on the possibility of vacuum decay via quantum tunnelling at a later epoch.

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